Math 170 Worksheet 6

1. Determine the set of points for which the following function is continuous.

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+xy+y^2} & \text{if } (x,y) \neq (0,0), \\ 0 & \text{if } (x,y) = (0,0). \end{cases}$$

2. Compute the first order partial derivatives of the functions below:

   (a) $f(x,y) = \dfrac{x}{y} - \dfrac{y}{x}$. (b) $f(\alpha,\beta) = (\alpha^2-\beta^3)^4$. (c) $f(s,t,r) = e^{st}\ln(sr)$.

3. Verify that $f_{xy} = f_{yx}$ for the following functions:

   (a) $f(x,y) = \dfrac{x^2}{x+y}$. (b) $f(x,y,z) = x^2\cos\left(\dfrac{y}{z}\right)$.

4. Show that $u(x,y) = e^{-x}\cos y + e^{-y}\cos x$ is harmonic, that is, $u_{xx} + u_{yy} = 0$.

5. Show that $w = \cos(x-y) + \ln(x+y)$ satisfies the partial differential equation

$$\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} = 0.$$

6. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if z is given implicitly as a function of x and y.

   (a) $x^2+y^2+z^2 = 3xyz$.

   (b) $yz = \ln(x+z)$.

   (c) $\sin(xyz) = x+2y+3z$.

Answers:

1. Continuous on $\mathbb{R}^2 \setminus \{(0,0)\}$. **2.** (a) $f_x = \dfrac{1}{y} + \dfrac{y}{x^2}$, $f_y = \dfrac{-x}{y^2} - \dfrac{1}{x}$. (b) $f_\alpha = 8\alpha(\alpha^2-\beta^3)^3$, $f_\beta = -12\beta^2(\alpha^2-\beta^3)^3$. (c) $f_s = te^{st}\ln(sr) + \dfrac{e^{st}}{s}$, $f_t = se^{st}\ln(sr)$, $f_r = \dfrac{e^{st}}{r}$.